Calculus Warm Up #1-3

Warm up sheets by the door.
Sit anywhere.

1. Fill in your name and Week 1 at the top of the page, then flip it over to the Wednesday space.

2. Take a minute to tell me if you have any concerns about this class or if you are called by a name other than that on my roster.

3. Get started on the function classwork activity worksheet. Work with your team.

Welcome to Calculus!

Your course syllabus can be found online at:

http://www.nicholsonsehs.wikispaces.com

What's next:

1) Some notes you will need for tonight's assignment

2) At 2:00, go down to the AV room and get your books.
(You will need student body card)

2.1 Introduction to Limits

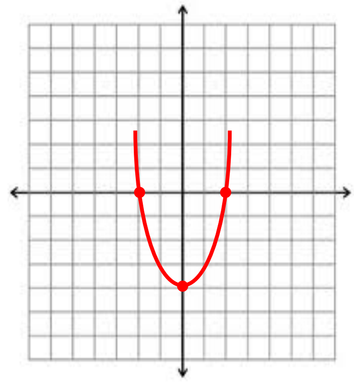
Informal definition of a limit: If f(x) becomes arbitrarily close to a unique number L as x approaches c from either side, then we say that the limit of f(x) as x approaches c, is L, and we write:

$$\lim_{x \to c} f(x) = L$$

What???

Example: $f(x) = x^2 - 4$

$f(2) = 4 - 4$
$= 0$



Consider:

$$\lim_{x \to c} f(x) = L$$

Looking at the graph:

for c = 2

$$\lim_{x \to 2} (x^2 - 4) = 0$$

Using a table to find limits: (go to 4 dec. pl.)

$$f(x) = \frac{x - 2}{x^2 - x - 2}$$

find: $\lim_{x \to 2} f(x)$

$$f(2) = \frac{2 - 2}{4 - 2 - 2} = \frac{0}{0}$$

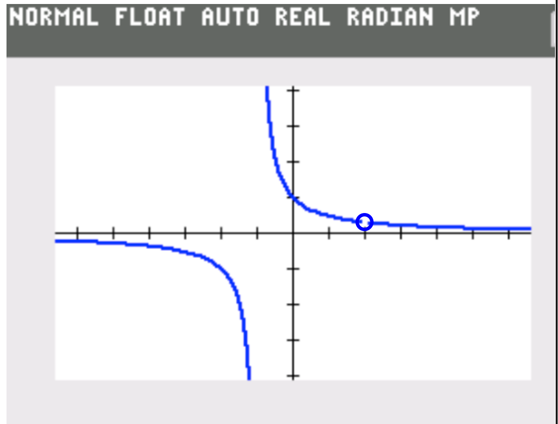
2

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

$$f(x) = \frac{x - 2}{x^2 - x - 2} = \frac{0}{0}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	.3448	.3344	.3334	.3332	.3322	.3226

f(2) is undefined, but we can find the limit as x approaches 2 from both the left side and the right side.



$$\lim_{x \to 2} f(x) = \frac{1}{3}$$

Another tool:

Factor and simplify the expression.

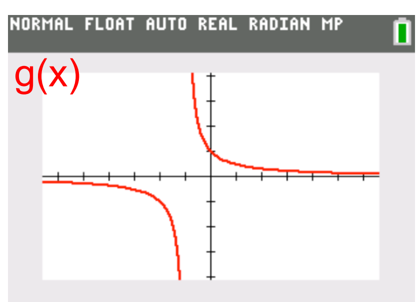
$$f(x) = \frac{x - 2}{x^2 - x - 2} = \frac{\cancel{x-2}}{\cancel{(x-2)}(x+1)}$$

$$g(x) = \frac{1}{x + 1}$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} g(x)$$

$$= \frac{1}{2 + 1}$$

$$= \frac{1}{3}$$



Limits we can find:

1. $\lim_{x \to 6} (2x - 1) = 2(6) - 1$
$= 11$

2. $\lim_{x \to 1} \frac{x - 1}{x^2 - 1} = \frac{1 - 1}{1^2 - 1} = \frac{0}{0}$ ☹ so try factor & cancel $\frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}}$

$= \lim_{x \to 1} \frac{1}{x+1} = \boxed{\frac{1}{2}}$

3. $\lim_{x \to 0} \frac{x}{\sqrt{x + 4} - 2} = \frac{0}{0}$ ☹ so Rewrite using the conjugate

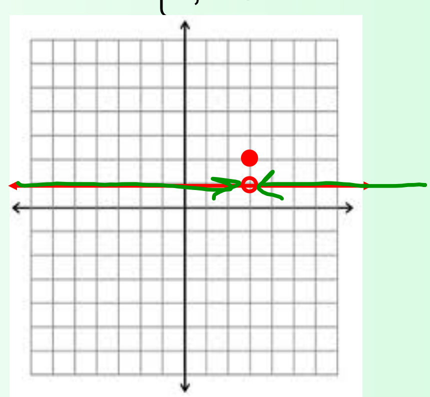
$\frac{x}{\sqrt{x+4} - 2} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}$

$= \lim_{x \to 0} \frac{x(\sqrt{x+4} + 2)}{x \cancel{+4} \cancel{-4}}$

$= \lim_{x \to 0} \frac{\cancel{x}(\sqrt{x+4} + 2)}{\cancel{x}} = \sqrt{0+4} + 2 = \boxed{4}$

4. $\lim_{x \to 3} f(x) =$

$f(x) = \begin{cases} 2, & x = 3 \\ 1, & x \neq 3 \end{cases}$



as x approaches 3 from both sides, the outcome of $f(x)$ is approaching $\boxed{1}$

even though $f(3) = 2$

☺

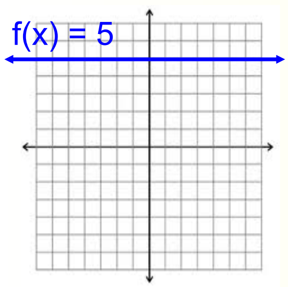
Basic Properties of Limits

If b and c are real numbers and n is a positive integer, then the following properties are true.

1. $\lim\limits_{x \to c} b = b$ 2. $\lim\limits_{x \to c} x = c$ 3. $\lim\limits_{x \to c} x^n = c^n$

Example:

$$\lim_{x \to 1} 5 = 5$$



#2 and #3 are just Direct Substitution

from Page 58:

Let b and c be real numbers, and n a positive integer, and let f and g be functions that each have a limit as $x \to c$.

1. Scalar multiple: $\lim\limits_{x \to c} [b\, f(x)] = b\left[\lim\limits_{x \to c} f(x)\right]$
2. Sum or difference: $\lim\limits_{x \to c} [f(x) \pm g(x)] = \lim\limits_{x \to c} f(x) \pm \lim\limits_{x \to c} g(x)$
3. Product: $\lim\limits_{x \to c} [f(x)g(x)] = \left[\lim\limits_{x \to c} f(x)\right]\left[\lim\limits_{x \to c} g(x)\right]$
4. Quotient: $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)}$, provided $\lim\limits_{x \to c} g(x) \neq 0$
5. Power: $\lim\limits_{x \to c} [f(x)]^n = \left[\lim\limits_{x \to c} f(x)\right]^n$

If $\lim_{x \to c} f(x) = 9$, find

a) $\lim_{x \to c} [f(x)]^2 = \left(\lim_{x \to c} f(x)\right)^2 = 9^2 = 81$

b) $\lim_{x \to c} [5f(x)] = 5\left(\lim_{x \to c} f(x)\right) = 5(9) = 45$

c) $\lim_{x \to c} [f(x)]^{\frac{5}{2}} = \left(\lim_{x \to c} f(x)\right)^{5/2}$

$= 9^{5/2}$

$= \left(\sqrt{9}\right)^5$

$= \boxed{243}$

Strategies to find limits

1. Look at a graph

2. Look at a table

3. Direct Substitution if the function is continuous

4. Factor and cancel to create a new function with an equivalent limit.

→ use conjugate

5. Rationalize the numerator or denominator

6. Find a common denominator to simplify compound fractions

One last trick: $$\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$$

Direct substitution gives you $\frac{0}{0}$, indeterminate.

Simplify the complex fraction and try again!

One last trick: $$\lim_{x \to 0} \frac{\frac{4}{4} \cdot \frac{1}{x+4} - \frac{1}{4} \cdot \frac{x+4}{x+4}}{x}$$

$$\frac{1}{x}\left[\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}\right]$$

$$\frac{1}{x}\left[\frac{-x}{4x+16}\right]$$

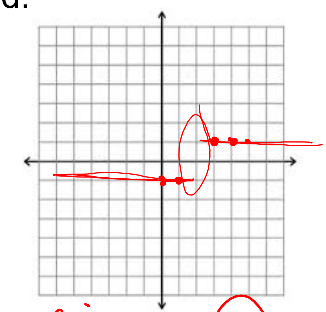
$$\lim_{x \to 0}\left(-\frac{1}{4x+16}\right) = -\frac{1}{16}$$

Examples of limits we can not find:

1. $\lim\limits_{x \to 2} \frac{|x-2|}{x-2} =$

DNE

x	y
0	-1
1	-1
3	1

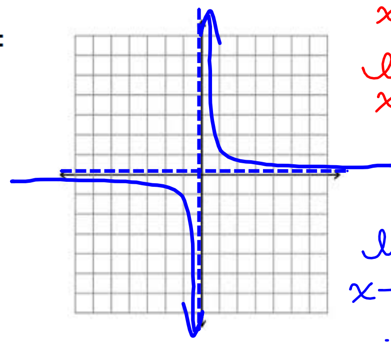


$\lim\limits_{x \to 2^+} = 1$

$\lim\limits_{x \to 2^-} = -1$

They don't match

2. $\lim\limits_{x \to 0} \frac{1}{x} =$



$\lim\limits_{x \to 0^+} = \infty$

$\lim\limits_{x \to 0^-} = -\infty$

limit DNE

HW: p. 61 #5 (to 4 dp), 7 - 12, 19, 25, 27, 29, 31, 32

p. 68 #1-9 odds, 17-23 odds

2:00, Go get your book!

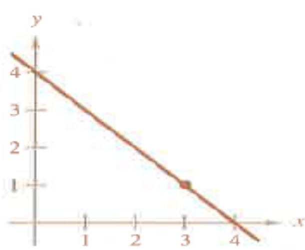
p. 61 #5, 7-12, 19, 25, 27, 29, 31

5. $\lim_{x \to 3} \dfrac{[1/(x+1)] - (1/4)}{x - 3}$

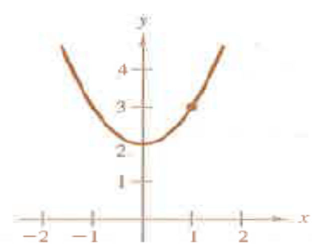
x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

In Exercises 7–12, use the given graph to find the limit (if it exists).

7. $\lim_{x \to 3} (4 - x)$

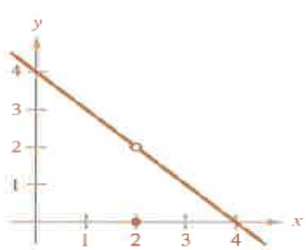


8. $\lim_{x \to 1} (x^2 + 2)$



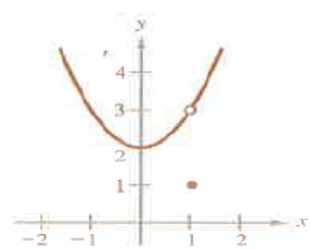
9. $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

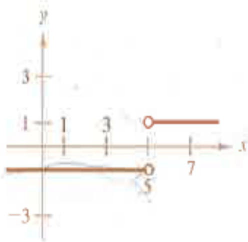


10. $\lim_{x \to 1} f(x)$

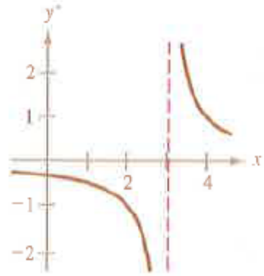
$$f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$$



11. $\lim_{x\to 5} \dfrac{|x-5|}{x-5}$



12. $\lim_{x\to 3} \dfrac{1}{x-3}$



Find the limit:

19. $\lim_{x\to 3} \sqrt{x+1}$

25. $\lim_{x\to -1} \dfrac{x^2+1}{x}$

27. If $\lim_{x\to c} f(x) = 2$ and $\lim_{x\to c} g(x) = 3$, find the following.

(a) $\lim_{x\to c} [5g(x)]$

(b) $\lim_{x\to c} [f(x) + g(x)]$

(c) $\lim_{x\to c} [f(x)g(x)]$

(d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$

29. If $\lim_{x\to c} f(x) = 4$, find the following.

(a) $\lim_{x\to c} [f(x)]^3$

(b) $\lim_{x\to c} \sqrt{f(x)}$

(c) $\lim_{x\to c} [3f(x)]$

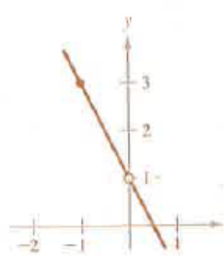
(d) $\lim_{x\to c} [f(x)]^{3/2}$

In Exercises 31 and 32, use a computer or graphics calculator to sketch the graph of the function f and find the specified limit (if it exists).

31. $f(x) = \dfrac{\sqrt{x+5} - 3}{x - 4}$, $\quad \lim\limits_{x \to 4} f(x)$

p. 68 #1-9 odds, 17-23 odds

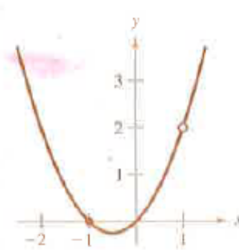
1. $g(x) = \dfrac{-2x^2 + x}{x}$



(a) $\lim\limits_{x \to 0} g(x)$

(b) $\lim\limits_{x \to -1} g(x)$

3. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim\limits_{x \to 1} g(x)$

(b) $\lim\limits_{x \to -1} g(x)$

5. $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

7. $\lim\limits_{x \to 3} \dfrac{x - 3}{x^2 - 9}$

9. $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

17. $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

19. $\lim\limits_{x \to 0} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

21. $\lim\limits_{x \to 0} \dfrac{[1/(2 + x)] - (1/2)}{x}$

23. $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 2} - \sqrt{2}}{x}$